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(54) **TRANSFUSIVE IMAGE MANIPULATION**

(71) Applicant: **DISNEY ENTERPRISES, Inc**, Burbank, CA (US)

(72) Inventors: **Alexander Sorkine-Hornung**, Zurich (CH); **Kaan Yucer**, Zurich (CH); **Alec Stefan Jacobson**, Zurich (CH); **Olga Sorkine-Hornung**, Zurich (CH)

(73) Assignee: **DISNEY ENTERPRISES, INC.**, Burbank, CA (US)

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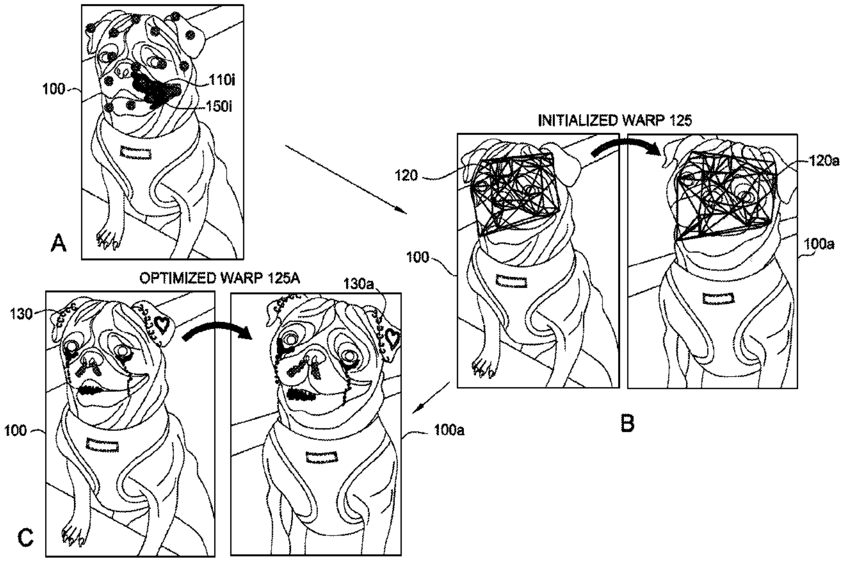
Primary Examiner — Stephen R Koziol
Assistant Examiner — Brian Kravitz
(74) *Attorney, Agent, or Firm* — Patterson & Sheridan, LLP

(57) **ABSTRACT**

The disclosure provides an approach for transferring image edits from a source image to target images. In one embodiment, a warp application receives a user-selected region of interest in a source image and determines for the region of interest content-aware bounded weight functions and seed locations for the same. For each of the target images, the warping application initializes a linear blend skinning subspace warp to a projection onto a feature space of a piecewise affine map from scale invariant feature transform features of the source image to the target image. After initializing the warps, the warping application iteratively optimizes the warps by applying the inverse compositional Lucas-Kanade procedure and using the content-aware weight functions in said procedure. Edits made to the source image may automatically be transferred to target images by warping those edits via the optimized warp function for the respective target images.

18 Claims, 4 Drawing Sheets



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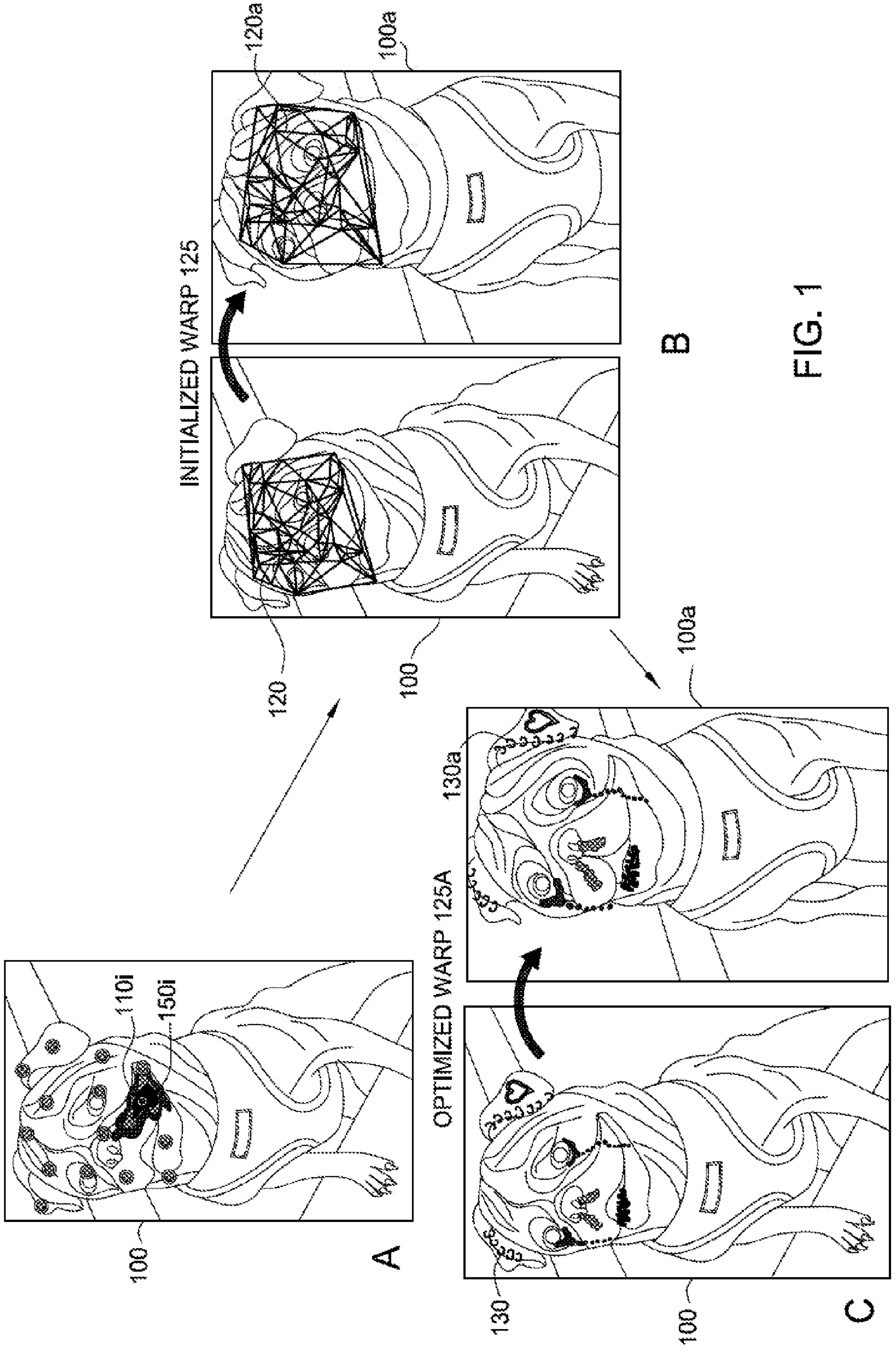
INTIALIZED WARP 125
120a
100a
120
100
B
100a
130a
OPTIMIZED WARP 125A
130
100
C
110i
150i
100
A

FIG. 1

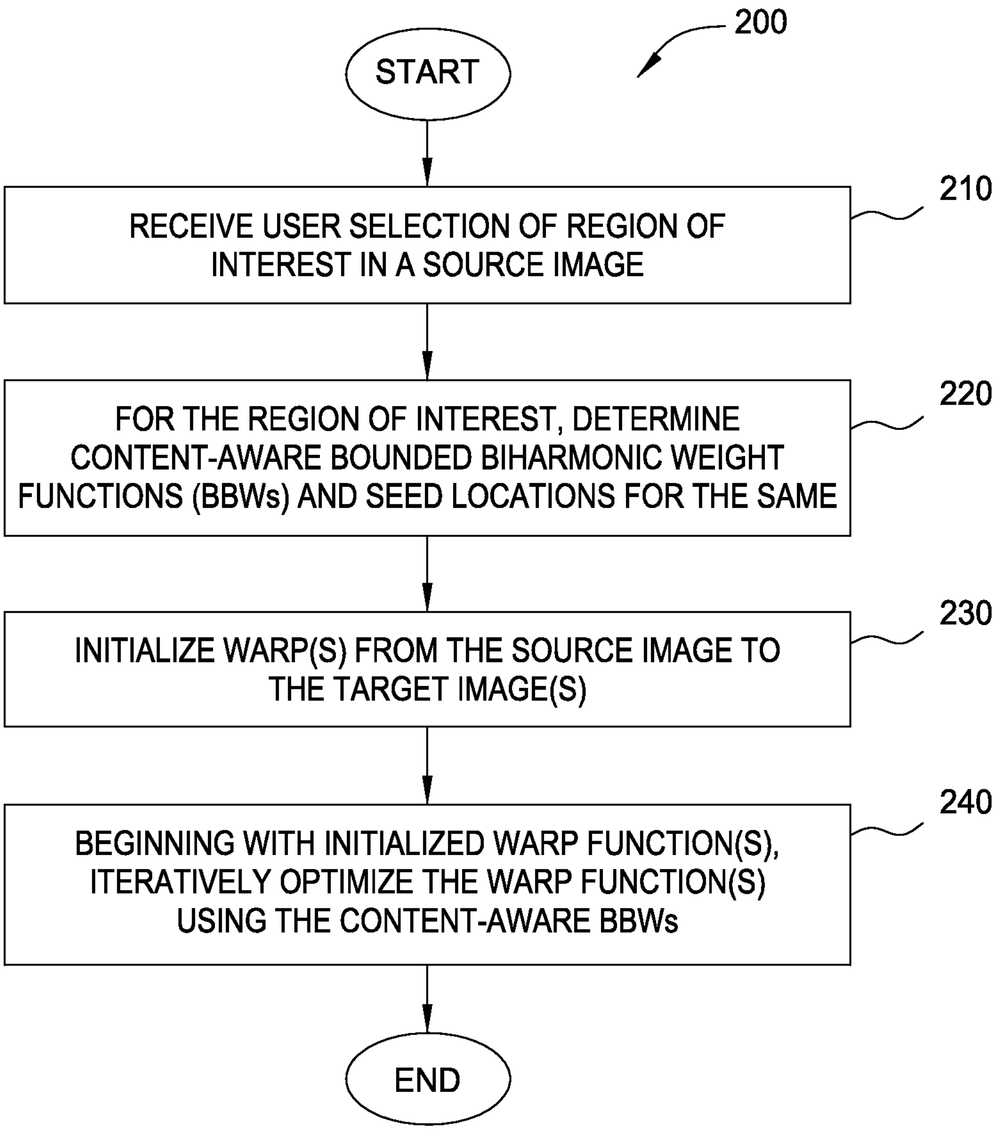
200
START
RECEIVE USER SELECTION OF REGION OF INTEREST IN A SOURCE IMAGE
210
FOR THE REGION OF INTEREST, DETERMINE CONTENT-AWARE BOUNDED BIHARMONIC WEIGHT FUNCTIONS (BBWs) AND SEED LOCATIONS FOR THE SAME
220
INITIALIZE WARP(S) FROM THE SOURCE IMAGE TO THE TARGET IMAGE(S)
230
BEGINNING WITH INITIALIZED WARP FUNCTION(S), ITERATIVELY OPTIMIZE THE WARP FUNCTION(S) USING THE CONTENT-AWARE BBWs
240
END

FIG. 2

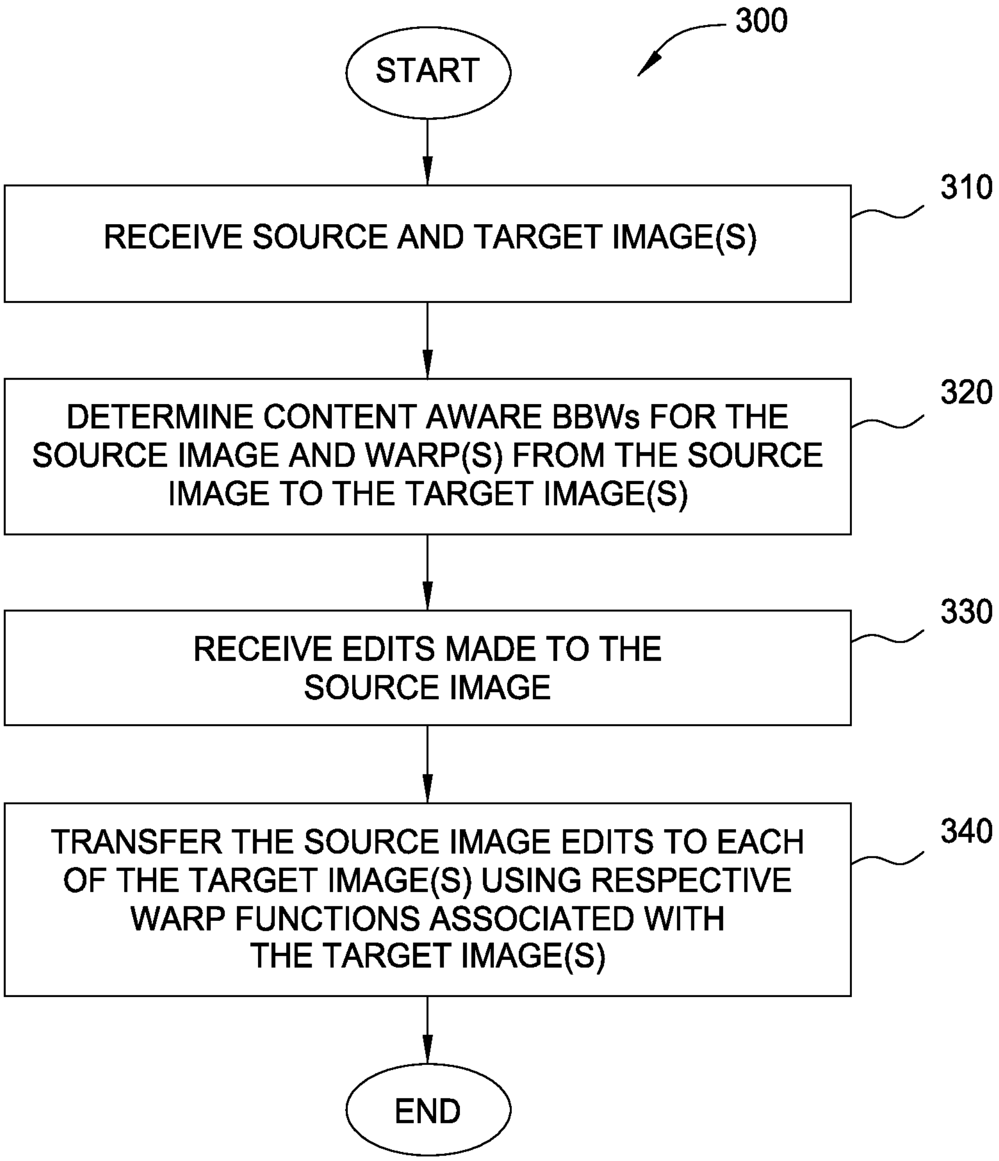
300
START
RECEIVE SOURCE AND TARGET IMAGE(S)
310
DETERMINE CONTENT AWARE BBWs FOR THE SOURCE IMAGE AND WARP(S) FROM THE SOURCE IMAGE TO THE TARGET IMAGE(S)
320
RECEIVE EDITS MADE TO THE SOURCE IMAGE
330
TRANSFER THE SOURCE IMAGE EDITS TO EACH OF THE TARGET IMAGE(S) USING RESPECTIVE WARP FUNCTIONS ASSOCIATED WITH THE TARGET IMAGE(S)
340
END

FIG. 3

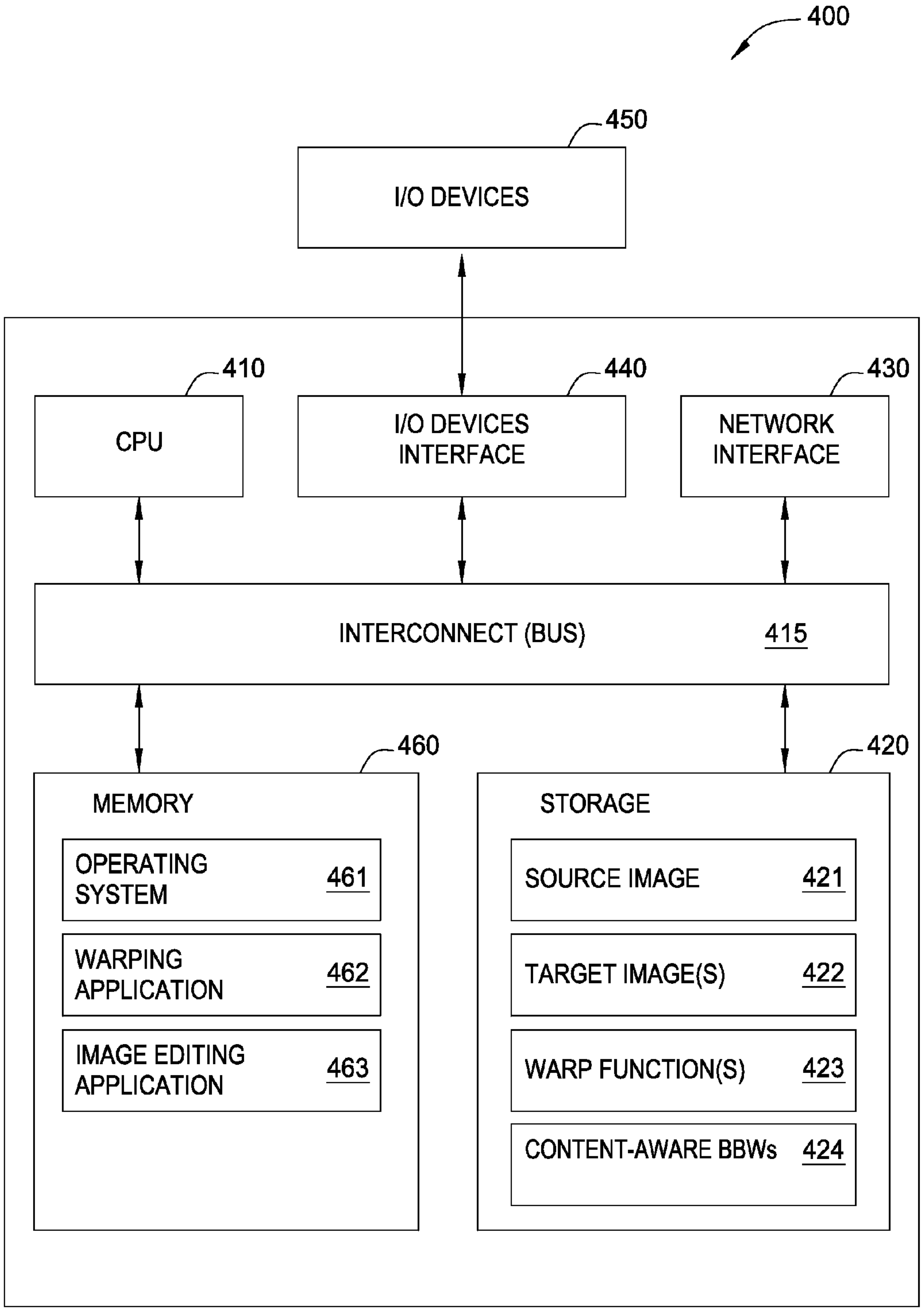
400
450
I/O DEVICES
410
CPU
440
I/O DEVICES INTERFACE
430
NETWORK INTERFACE
INTERCONNECT (BUS) 415
460
MEMORY
OPERATING SYSTEM 461
WARPING APPLICATION 462
IMAGE EDITING APPLICATION 463
420
STORAGE
SOURCE IMAGE 421
TARGET IMAGE(S) 422
WARP FUNCTION(S) 423
CONTENT-AWARE BBWs 424

FIG. 4

TRANSFUSIVE IMAGE MANIPULATION

BACKGROUND

1. Field

Embodiments presented herein provide techniques for image manipulation. More specifically, embodiments presented herein disclose techniques for transferring edits made to one image to other images of the same object or scene.

2. Description of the Related Art

Transferring pixel-level edits from one image to another requires accurate mappings between corresponding image regions. General purpose correspondence estimation techniques include dense optical flow. However, such techniques are akin to cloning and can only handle small image displacements and appearance changes. That is, such approaches do not permit edits made to a source image to be correctly transferred to target images which differ from the source image as a result of large perspective changes, illumination changes, or non-rigid deformations. Further, such approaches do not account for relatively large occlusions in the target image(s). Edits cannot be correctly transferred from the source image if the target image(s) have such occlusions.

SUMMARY

One embodiment of the invention includes a computer implemented method for transferring image edits from a source image to a target image. The method includes determining, via one or more processors, content-aware weights for the source image. The method further includes determining, via one or more processors, an initial warp from the source image to the target image, and iteratively optimizing the initial warp using the content-aware weights. In addition, the method includes applying the optimized warp to transfer the image edits to the target image.

Other embodiments include a computer-readable medium that includes instructions that enable a processing unit to implement one or more aspects of the disclosed methods as well as a system configured to implement one or more aspects of the disclosed methods.

BRIEF DESCRIPTION OF THE DRAWINGS

So that the manner in which the above recited aspects are attained and can be understood in detail, a more particular description of aspects of the invention, briefly summarized above, may be had by reference to the appended drawings.

It is to be noted, however, that the appended drawings illustrate only typical aspects of this invention and are therefore not to be considered limiting of its scope, for the invention may admit to other equally effective aspects.

FIG. **1** illustrates an approach for transferring image edits, according to an embodiment of the invention.

FIG. **2** illustrates a method for determining a warp function for transferring edits from a source image to target images, according to an embodiment of the invention.

FIG. **3** illustrates a method for transferring image edits made to a source image to target images, according to an embodiment of the invention.

FIG. **4** depicts a block diagram of a system in which an embodiment may be implemented.

DETAILED DESCRIPTION

Embodiments disclosed herein provide techniques for transferring image edits from a source image to target images.

The techniques include receiving, by a warping application, a user-selected region of interest in a source image and determining for the region of interest content-aware weight functions, such as content-aware bounded biharmonic weights (BBWs), and seed locations for the same. Here, content-aware BBWs may be obtained via BBW optimization in a feature space such as the International Commission on Illumination (CIE) L*a*b* color space (also referred to herein as "L*a*b* color space"). Such optimization produces weights sensitive to image content and propagating to similar image areas while dropping off at salient edges, among other properties. For each target image, the warping application initializes a warp (also referred to herein as "warp function") which takes respective source image pixel coordinates and maps them to pixels coordinates of the target image. In one embodiment, each warp function may be a warp in linear blend skinning subspace of deformations, and each warp may be initialized to a projection onto a feature space of a piecewise affine map between scale invariant feature transform features of the source image and the respective target image. After initializing the warps, the warping application iteratively optimizes the warps by applying an inverse compositional Lucas-Kanade procedure and using content-aware BBWs in said procedure. Thereafter, edits made to the source image may automatically be transferred to target images by warping those edits via the optimized warp functions for the respective target images.

The following description references aspects of the disclosure. However, it should be understood that the disclosure is not limited to specific described aspects. Instead, any combination of the following features and elements, whether related to different aspects or not, is contemplated to implement and practice the disclosure. Furthermore, although aspects of the disclosure may achieve advantages over other possible solutions and/or over the prior art, whether or not a particular advantage is achieved by a given aspect is not limiting of the disclosure. Thus, the following aspects, features, and advantages are merely illustrative and are not considered elements or limitations of the appended claims except where explicitly recited in a claim(s). Likewise, reference to "the disclosure" shall not be construed as a generalization of any inventive subject matter disclosed herein and shall not be considered to be an element or limitation of the appended claims except where explicitly recited in a claim(s).

Aspects of the present disclosure may be embodied as a system, method or computer program product. Accordingly, aspects of the present disclosure may take the form of an entirely hardware aspect, an entirely software aspect (including firmware, resident software, micro-code, etc.) or an aspect combining software and hardware aspects that may all generally be referred to herein as a "circuit," "module" or "system." Furthermore, aspects of the present disclosure may take the form of a computer program product embodied in one or more computer readable medium(s) having computer readable program code embodied thereon.

Any combination of one or more computer readable medium(s) may be utilized. The computer readable medium may be a computer readable signal medium or a computer readable storage medium. A computer readable storage medium may be, for example, but not limited to, an electronic, magnetic, optical, electromagnetic, infrared, or semiconductor system, apparatus, or device, or any suitable combination of the foregoing. More specific examples (a non-exhaustive list) of the computer readable storage medium would include the following: an electrical connection having one or more wires, a portable computer diskette, a hard disk, a random access memory (RAM), a read-only memory (ROM), an erasable programmable read-only memory (EPROM or Flash memory), an optical fiber, a portable compact disc read-only memory (CD-ROM), an optical storage device, a magnetic storage device, or any suitable combination of the foregoing. In the context of this document, a computer readable storage medium may be any tangible medium that can contain, or store a program for use by or in connection with an instruction execution system, apparatus or device.

The flowchart and block diagrams in the Figures illustrate the architecture, functionality and operation of possible implementations of systems, methods and computer program products according to various aspects of the present disclosure. In this regard, each block in the flowchart or block diagrams may represent a module, segment or portion of code, which comprises one or more executable instructions for implementing the specified logical function(s). In some alternative implementations the functions noted in the block may occur out of the order noted in the figures. For example, two blocks shown in succession may, in fact, be executed substantially concurrently, or the blocks may sometimes be executed in the reverse order, depending upon the functionality involved. Each block of the block diagrams and/or flowchart illustrations, and combinations of blocks in the block diagrams and/or flowchart illustrations can be implemented by special-purpose hardware-based systems that perform the specified functions or acts, or combinations of special purpose hardware and computer instructions.

FIG. **1** illustrates an approach for transferring image edits, according to an embodiment of the invention. As shown in panel A, content-aware bounded biharmonic weights (BBWs) (e.g., content-aware bounded BBW **110***i*) are determined for a source image **100**. Each content-aware BBW controls a region of the image **100** (e.g., content-aware BBW **110***i* controls region **150***i*) in the sense that pixels in the region are translated, rotated, squeezed, sheered, and the like based on how much those pixels are influenced by the controlling BBW. Further, each content-aware BBW is "content aware" in that the weights are sensitive to content of the image and propagate to similar image areas, while dropping off at salient edges which may correspond to boundaries of objects, boundaries of surface markings, discontinuities in surface orientation, and the like. The content-aware BBWs may be used while iteratively optimizing a warp **125** from the source image **100** to the target image **100***a*. The optimized warp **125***a* may then be applied to transfer edits from the source image **100** to the target image **100***a*.

As discussed in greater detail below, the content-aware BBWs may be determined by optimizing BBWs in a feature space, such as the International Commission on Illumination (CIE) L*a*b* color space. Seed locations for the content-aware BBWs may be determined by, e.g., applying an operator to the source image to determine the magnitudes of the intensity gradient (dI/dx, dI/dy) (i.e., the gradient magnitude values) in the source image, creating a confidence image itself created by applying heat diffusion to gradient magnitude values, and then iteratively selecting a number of highest confidence locations while updating the confidence image after each selection by subtracting a Gaussian kernel.

As shown in panel B, a warp from the source image **100** to the target image **100***a* is computed by iterative optimization, starting from an initialized warp **125** which is a projection of a map between scale-invariant transform (SIFT) features **120**, **120***a* of the source and target images **100**, **100***a*, respectively, to a space of maps spanned by the degrees of freedom of the feature space. The warp **125** is a function which receives pixel coordinates in the source image **100** as input and outputs corresponding coordinates for the target image **100***a*. That is, the warp **125** provides an image registration between region(s) of the source and target images. For example, the warp **125** may be used to deform the source image **100** into an image which is a close match to the target image **100***a*. Given an edit made to the source image **100**, the edit may be plugged into the warp function **125** to determine a corresponding edit for the target image **100***a*. In one embodiment, the initialized warp function **125** may be iteratively optimized according to an "inverse compositional" Lucas-Kanade procedure, thereby producing an optimized warp **125***a* from the source image **100** to the target image **100***a*. The inverse compositional Lucas-Kanade procedure is a gradient-based iterative optimization technique which may be used to optimize parameters so as to reduce a mismatch between two images. Of course other iterative optimization procedures may be used. Further, the content-aware BBWs may be used in the Lucas-Kanade (or other iterative optimization) procedure, as discussed in greater detail below.

As shown in panel C, image edits **130** made to the source image **100** are transferred to the target image **100***a* via the optimized warp **125***a*. In general, the image edits **130** may include any user made or automatically generated alterations to the source image **100**. Illustratively, various circles, hearts, and other edits have been drawn onto the dog depicted in the source image **100**. The edits **130**, when transferred to the target image **100***a*, appear as edits **130***a*. To obtain the edits **130***a*, coordinates in the source image **100** associated with the edits **130** may simply be plugged into the optimized warp function **125***a*, which then outputs coordinates (e.g., floating point coordinates corresponding to sub-pixel locations) for those edits in the target image **100***a*. The edits **130***a* may then be made at the appropriate locations in the target image **100***a*. For example, as image edits are made to the source image **100**, the edits may be automatically transferred to the target image **100***a* using the optimized warp function **125***a*. Of course, such image edits may also be transferred to the target image **100***a* at a later time.

FIG. **2** illustrates a method **200** for determining a warp function for transferring edits from a source image I_s to target images I_t, t=1, . . . , N, according to an embodiment. As shown, the method **200** begins at step **210**, where a warping application receives a user selected region of interest in the source image. The goal being to transfer edits from the source image I_s to target images I_t, t=1, . . . , N, a warping application may determine an optimal map $\mathcal{M}_t$ which takes pixel coordinates $p \in I_s$ to new pixel coordinates $p' \in I_t$ (or, in some embodiments, sub-pixel coordinates). In general, maps may be considered over the entire source domain, but, because edits may only be made to a subregion of the source image $R_s \subseteq I_s$, the user may select such a region as the region of interest so that the warping application only considers warps over the region. Doing so may permit faster and better convergence during iterative optimization, discussed below, because the warping application may ignore extraneous regions of the source image on which edits are not made. In one embodiment, the region of interest may be fuzzy, defined by the mask function $r(p) \in [0,1]$ which reaches 1 at pixels fully inside the region and fades to 0 at the boundary of R_s.

At step **220**, the warping application determines, for the user-selected region of interest, content-aware bounded biharmonic weight functions (BBWs) and seed locations for the same. Determining the optimal map $\mathcal{M}_t$ by optimizing over all possible maps is typically intractable. Instead, a subspace of warp functions spanned by the linear combination of a number of affine transformations may be used, such as the linear blend skinning (LBS) subspace of deformations:

$$\mathcal{M}(p)=\Sigma_{k=1}^{m} w_k(p) T_k p \qquad (1)$$

Here, w_k are spatially-varying scalar weight functions, and T_k are affine transformations which may be optimized. When carefully designing the minimization process and the weight functions w_k, the LBS subspace can be sufficiently expressive, producing accurate and intuitive warps which also balance between smoothness and content-adaptiveness. In particular, the weight functions w_k may be defined to provide parametrizable smoothness and content-adaptivity by interpreting images as manifolds, as discussed below. Experience has shown that working in the LBS subspace is advantageous because the subspace provides a controllable balance between computationally expense and expressiveness and accuracy of the mapping. Despite optimizing only the parameters of a few affine transformations, the content-adaptive weighted interpolation allows for mapping functions that faithfully follow the shape of even complex objects. In addition, the reduced subspace acts as a regularization which avoids the pitfalls and local minima issues found in optical flow techniques. This allows matching of significant changes in viewpoint, illumination, non-rigid deformation, and occlusions so that edits may be accurately transferred from the source image to the target images.

Classically, BBWs have been used as weight functions for realtime deformation of 2D and 3D shapes. Typically, each classical BBW w_k is associated with a handle H_k which is a region (or a single point) in the domain fully controlled by that weight function. As a result, w_k has value 1 on H_k and 0 on all other handles. The weight function w_k may be defined by optimizing Laplacian energy subject to constant bound constraints:

$$\arg\min_{w_k,k=1,\ldots,m}\Sigma_{k=1}^{m}\tfrac{1}{2}\int_{l_s}(\Delta w_k)^2 dx\,dy \quad (2)$$

$$\text{subject to: } w_k|H_l=\delta_{kl},k,l=1,\ldots,m \quad (3)$$

$$\Sigma_{k=1}^{m}w_k(p)=1\forall p\in l_s \quad (4)$$

$$0\le w_k(p)\le 1,k=1,\ldots,m,\forall p\in l_s. \quad (5)$$

Assuming the 2D region of interest domain R_s is discretized with a triangle mesh Ω with n vertices, the discrete optimization problem for weight functions $w_k\in\mathbb{R}^n$ may be written as follows:

$$\arg\min_{w_k}\Sigma_{k=1}^{m}w_k^TQw_k, \quad (6)$$

subject to the constraints (3)-(5). The coefficients matrix of the energy is the discrete biharmonic operator $Q=LM^{-1}L$, where the matrices L and M are the stiffness and diagonal mass matrices, respectively, of the discretization mesh Ω:

$$L_{ij}=\begin{cases}-\frac{1}{2}(\cot\alpha_{ij}+\cot\alpha_{ji}) & \text{if } j\in N(i)\\ -\sum_{l\in N(i)}L_{il} & \text{if } j=1\\ 0 & \text{otherwise}\end{cases} \quad (7)$$

$$M_{ii}=\text{Voronoi area around vertex } i, \quad (8)$$

where N(i) is the set of mesh neighbors of vertex i, and α_{ij} is the angle opposite the directed edge from i to j on the incident triangle lying to the left of the edge (if it exists).

The weights w_k resulting from the foregoing optimization enjoy smoothness guarantees of a fourth-order energy minimizer, as well as locality, boundedness, and interpolation. However, such BBW weights are not content aware, because their definitions depend solely on the handle locations H_k and the Riemannian metric of the domain R_s imposed by the biharmonic operator.

The classical weight functions discussed above may be modified so as to adapt them to image content. In particular, the geometry of the discretization mesh Ω may be mapped into a higher-dimensional feature space $\mathbb{R}^D$, such that the mesh becomes a 2-manifold in that space. For each vertex v∈Ω, some of the attributes of the vertex may be concatenated to the 2D spatial coordinates of the vertex as additional dimensions. For example, the following 5-dimensional embedding may be obtained by expressing image colors at the vertices in CIE 1976 (L*, a*, b*) color space:

$$v(x,y)\rightarrow(x,y,s_l l(x,y),s_a a(x,y),s_b b(x,y)), \quad (9)$$

where the scaling factors s* account for color value coordinates not being readily proportional to Euclidean pixels and further allow for varying the sensitivity of the weights to image content. In one embodiment, s* may be taken to be 16.

The following properties may be obtained by optimizing BBW weights over the 2-manifold in feature space: (i) sensitivity to image content and propagation to similar image areas, dropping off at salient edges; (2) localized (i.e., have limited spatial interaction range); (3) smooth (at least C^1) everywhere such that warping is smooth in smooth image regions; and (4) limited to the [0,1] range. For example, in the case of the L*a*b* color space discussed above, strong image edges will become steep creases in a 5D 2-manifold, making the travel distance on the manifold across the edge long, such that the weight function will have an abrupt derivative across the edge. Further, experience has shown that L*a*b* color space works better than some other feature spaces (e.g., grayscale and RGB feature spaces) in obtaining the foregoing properties.

To define the BBW optimization problem in feature space, the discretization of the bi-Laplacian operator $Q=LM^{-1}L$ may be adjusted to the Riemmanian metric of the image domain embedded in the $\mathbb{R}^D$ feature space. In particular, the stiffness and diagonal mass matrices L and M may be adapted to have the same formulas of cotangents and area, but of mesh triangles embedded in $\mathbb{R}^D$.

In one embodiment, the lengths l_{ij}, l_{jk}, l_{ki} of the three sides of a triangle T_{ijk} may be used to extract the necessary quantities, as lengths are easy to compute in any dimension. The triangle area A_{ijk} is given by Heron's formula:

$$A_{ijk}=\sqrt{r(r-l_{ij})(r-l_{jk})(r-l_{ki})}, \quad (10)$$

where r is the semi-perimeter $\tfrac{1}{2}(l_{ij}+l_{jk}+l_{ki})$. Cotangents of the given angles may be determined via trigonometric identities as:

$$\cot(\alpha_{ij})=\frac{l_{jk}^2+l_{ki}^2-l_{ij}^2}{4A_{ijk}},\quad \cot(\alpha_{jk})=\frac{l_{ki}^2+l_{ij}^2-l_{jk}^2}{4A_{ijk}},\quad \cot(\alpha_{ij})=\frac{l_{ij}^2+l_{jk}^2-l_{ki}^2}{4A_{ijk}} \quad (11)$$

Such cotangents may be used to construct and solve the BBW optimization problem for the weights w_k over the surface in $\mathbb{R}^D$. In particular, such optimization amounts to sparse quadratic programming (QP) with constant inequality constraints. As a result, the weights w_k may be determined using, for example, MOSEK solver.

In one embodiment, each weight function w_k may be solved for separately, dropping the partition of unity constraint of equation (4) and then normalizing the weights to sum to 1. In a further embodiment, the source image may be downsampled before solving for weight functions w_k, and the weight functions w_k may later be upsampled to full resolution using interpolation, such as bicubic interpolation. Doing so avoids using the full-resolution source image, because solving for weight functions w_k with the fine resolution of source images may be computationally expensive. Advantageously, content-aware weight functions, as described herein, are resilient to changes in discretization resolution. While downsampling too much may have a smoothing effect and change the support regions of each weight function, the values and supports of the weight functions tend to change little as discretion approaches full resolution.

In the classical formulation of BBWs, the weight functions are meant to be attached to user-specified handles H_k, which are placed by the user onto regions that she intends to control. In contrast, content-aware BBWs may be automatically placed at seeds locations where the weights are respectively constrained to 1 and 0. In particular, because content-aware BBWs tend to fall off at salient edges, one embodiment may choose seed locations in a manner that approximates uniform distribution over the region of interest R_s and place the BBWs away from strong edges. In such a case, the warping application may determine a gradient magnitude of the source image and apply head diffusion on the gradient magnitude values to create a confidence image. Then, the warping application may iteratively select the m highest confident locations, updating the confidence map after each selection by subtracting a Gaussian kernel with standard deviation $\sqrt{|R_s|/(\pi m)}$. Advantageously, generating seed locations using this approach is simple and computationally efficient. However, various other distribution methods may also be used to generate seed locations.

At step **230**, the warping application initializes warp(s) from the source image to the target image(s). Doing so provides a starting point for iterative optimization of the warp function(s), discussed in greater detail below. In one embodiment, iterative optimization may begin with an initial warp $\mathcal{M}^0$ parametrized by initial affine transformations T_k^0. Here, trivial initial guesses (e.g., using the identity transformation for both $\mathcal{M}^0$ and T_k^0) may result in slow convergence due to the nonlinear nature of the energy. To alleviate such slow convergence, the warping application may use a sparse set of scale-invariant feature transform (SIFT) features as the initial guess.

SIFT features in the source and target images may be determined using a variety of methods, such as that discussed in Lowe, D. G. 2004, Distinctive image features from scale-invariant keypoints, *Int. J. Comput. Vision* 60, 2, 91-110, hereby incorporated by reference in its entirety. Further, correspondences between the SIFT features in the source and target images may be determined via, e.g., Random Sample Consensus (RANSAC). In the ideal case, there may be many correct matches all over the region of interest R_s. The Delaunay triangulation τ of the source feature locations and the mappings of this triangulation to the matched target feature locations to define a piecewise-affine map $\mathcal{M}_{SIFT}$ for each of the target image(s). In extreme cases where there are too few matches, one may manually add corresponding points to the source and target images. Note, neither the SIFT features nor the optional manual correspondences are hard constraints (they are used only for initialization purposes), so they need not be particularly accurate.

For each of the target image(s), the piecewise affine map $\mathcal{M}_{SIFT}$ may be projected to the space of maps spanned by the appropriate degrees of freedom. That is, the warping application may determine the initial affine transformations T_k^0 that best reproduce $\mathcal{M}_{SIFT}$ in a least-squares sense

$$\arg\min_{T_k^0,k=1,\ldots,m}\Sigma_{p\in\tau}r(p)\|\mathcal{M}_{SIFT}(p)-\Sigma_{i=1}^{m}w_k(p)T_k^0p\|^2, \tag{12}$$

which amounts to solving a dense linear system with 6m variables.

If SIFT features are only found in one part of the source image, the above system may be undetermined, as there may exist transformations T_k whose corresponding weight functions w_k are zero or near zero for all pixels in τ. In such cases, the warping application may regularize the weight functions w_k against the best-fit global affine transformation A found using RANSAC on the SIFT features, adding the following term to the minimization of equation (12):

$$\gamma\Sigma_{p\in l_s}r(p)\|Ap-\Sigma_{i=1}^{m}w_k(p)T_k^0p\|^2, \tag{13}$$

where γ balances between fitting to $\mathcal{M}_{SIFT}$ inside T and matching A elsewhere.

Although described above as a fully automated process, alternative embodiments may allow a user to assist in the initialization process. For example, the user may provide a set of points for each target image which corresponds to a set of points of the source image. Such user assistance may help the iterative optimization, discussed below, to converge in some cases.

At step **240**, the warping application iteratively optimizes the warp(s) from the source image to the target image(s), beginning with the initialized warp determined at step **230**, using the content-aware BBWs determined at step **220**. In one embodiment, the warping application may perform iterative optimization using the Lucas-Kanade (LK) procedure. In particular, the LK procedure and its extensions may be used to align corresponding image regions, thereby permitting a consistent transfer of edits between images. The basic LK procedure computes parameters of a warp function $\mathcal{M}$ by an iterative minimization of the color mismatch between source image pixels I_s (p) and corresponding warped pixels in some target image $I_t\mathcal{M}$ (p)):

$$\arg\min_{\mathcal{M}}\Sigma_{p\in l_s}r(p)\phi(I_t(M(p)),I_s(p)). \tag{14}$$

Here, ϕ is an error norm which measures the difference in color values between a source pixel p and the corresponding pixel $\mathcal{M}_t$(p). For ϕ, a robust error norm may be used for increased robustness to image consistencies (e.g., occlusions). For example, one embodiment may use adaptive thresholding with spatial coherence approximation. In addition, linear appearance changes may be accounted for within the same framework to compensate for changes in illumination during the matching, as discussed in greater detail below.

In one embodiment, the search space of the optimization may be reduced by considering only the subspace of warps $\mathcal{M}$ defined by weighted linear combinations of a small number of affine transformations. If the per-pixel weight functions w_k are precomputed, then the only parameters of such a subspace are the elements of the affine transformation matrices $T_k\in\mathbb{R}^{2\times3}$. That is, given m pairs of weight functions and transformations, the subspace is parametrized by only 6m degrees of freedom. In such a case, the optimization becomes:

$$\arg\min_{T_k,k=1,\ldots,m}\Sigma_{p\in l_s}r(p)\phi(I_t(\Sigma_{i=1}^{m}w_k(p)T_kp),l_s(p)). \tag{15}$$

Warp functions $\mathcal{M}$ according to equation (15) may readily be integrated into the standard "forward additive" variant of the LK procedure. Experience has shown, however, that such a variant of the LK procedure may be computationally inefficient, as it requires a re-computation of the Hessian matrix at each iteration of the algorithm. In one embodiment, a modification of the more computationally-efficient "inverse compositional" variant of LK to search over the parameterized space.

In the "inverse compositional" LK, an incremental warp update is utilized in which the roles of I_s and I_t are exchanged compared to in equation (15). This enables more efficient implementations, as the Hessian and other expensive steps can be precomputed. However, a global warp must be iteratively composed with the following update rule:

$$\mathcal{M}(p) \leftarrow \mathcal{M}(p) \circ \Delta\mathcal{M}(p)^{-1}, \tag{16}$$

where $\Delta\mathcal{M}$ is a warp update computed in every iteration. Inserting equation (1) into equation (16) gives:

$$\mathcal{M}(p) \leftarrow [\Sigma_{k=1}^{m} w_k(p) T_k p][\Sigma_{l=1}^{m} w_l(p) \Delta T_l p]^{-1}, \tag{17}$$

where ΔT_k are updates to the affine transformation matrices T_k. Equation (17) may be rewritten using the distributive property as:

$$= \Sigma_{k=1}^{m} [w_k(p) T_k [\Sigma_{l=1}^{m} w_l(p) T_l]^{-1}] p. \tag{18}$$

Given BBWs which are sufficiently content-aware, localized, and smooth (e.g., content-aware BBWs), ΔT_l may be replaced by ΔT_k in the inner sum of equation (18), resulting in:

$$\approx \Sigma_{k=1}^{m} [w_k(p) T_k [\Sigma_{l=1}^{m} w_l(p) \Delta T_k]^{-1}] p. \tag{19}$$

The reasoning behind such an approximation will be apparent from a consideration of the influence of a transformation and weight function pair (T_k and w_k) on some pixel p. Here, it is instructive to distinguish between three cases.

In the first case, pixel p is dominated by the weight function w_k, i.e., $\forall l \neq k$: $w_l(p) \approx 0$. In such a case, the influences of all other transformations ΔT_l with $l \neq k$ on p are negligible and only the contribution for l=k remains in the inner sum.

In the second case, pixel p has a low weight $w_k(p) \approx 0$. Consequently, neither the associated transformation T_k nor the inner sum have a significant influence on p and may be neglected.

In the third case, pixel p is influenced by multiple constraints, i.e., $w_k(p) > 0$ for some set K of k's. Given content-aware weights, the weights should fall off quickly near edges. As a result, pixel p may be influenced by multiple constraints only when it is located within a smoothly varying image region in I_s. Further, it is reasonable to assume that the corresponding region in I_t is also smoothly varying. By replacing ΔT_l with ΔT_k, $k \in K$ in the inner sum of equation (18), the assumption is made that, in such smoothly varying image regions, constraints with overlapping weight functions undergo similar transformations between images.

Assuming that the weight functions partition unity at every pixel, equation (19) can further be simplified to:

$$\mathcal{M}(p) \leftarrow \Sigma_{k=1}^{m} [w_k(p) T_k [\Delta T_k]^{-1}] p. \tag{20}$$

Here, the inner sum of weighted affine transformations has been reduced to a single, easily-inverted transform. Experience has shown that, despite the approximation made in equation (20), the solution of equation (20) robustly and efficiently converges to a pixel-accurate matching even in challenging cases.

In one embodiment, in addition to the robust error norm ϕ discussed above, two additional degrees of freedom may be used to account for global bias and gain. In general, different pixels will undergo different appearance changes, so ideally bias and gain parameters would be per-pixel. However, such an approach may be computationally expensive. Experience has shown that content-aware BBW weights may be used instead to blend appearance parameters. In particular, smooth lighting variations, as well as discontinuities in illumination, can be handled using such weights. In practice, this can be achieved by replacing $I_s(p)$ in equation (15) with

$$I_s(p) + \Sigma_{k=1}^{m} w_k(p)(a_k I_s(p) + b_k), \tag{21}$$

where a_k and b_k represent the bias and gain corresponding to weight w_k, and determining new bias and gain degrees of freedom during the iterative optimization discussed above.

FIG. **3** illustrates a method **300** for transferring image edits made to a source image to target images, according to an embodiment. As shown, the method **300** begins at step **310**, where a warping application receives source and target image(s). At step **320**, the warping application determines content-aware BBWs for the source image and warp(s) from the source image to each of the target image(s). In one embodiment, the warping application may determine content-aware BBWs by optimizing a BBW in feature space such as the L*a*b* color space, as discussed above. Further, based on the determined content-aware BBWs, the warping application may determine warp(s) from each of the target image(s) to the source image by iterative optimization using the inverse compositional LK procedure, similar to the discussion above with respect to FIG. **2**.

At step **330**, the warping application receives edits made to the source image. Here, edits may include changes to any part(s) of the source image (e.g., changes in color, brightness, hue, contrast, etc. to part(s) of the source image). Such edits may be made via a separate image editing application, such as Adobe's Photoshop®, or the warping application itself may include a component which allows users to edit images.

At step **340**, the warping application transfers the source image edits to each of the target image(s) by applying the respective warp function(s) associated with those images. That is, the warping application determines, using the respective warp function(s), pixel (or sub-pixel) location(s) in the target image(s) which correspond to the pixel location(s) of the source image edits, and applies appropriate edits corresponding to those made to the source image at the determined pixel (or sub-pixel) location(s) of the target image(s). In one embodiment, edit transfers may be made in real time as image edits are made by a user. Of course, the edit transfers may also be made at a later time.

FIG. **4** illustrates a system **400** in which aspects of the invention may be implemented. As shown, the system **400** includes, without limitation, a central processing unit (CPU) **410**, a network interface **430**, an interconnect **415**, a memory **460** and storage **420**. The system **400** may also include an I/O device interface **440** connecting I/O devices **450** (e.g., keyboard, display and mouse devices) to the system **400**.

The CPU **410** retrieves and executes programming instructions stored in the memory **460**. Similarly, the CPU **410** stores and retrieves application data residing in the memory **460**. The interconnect **415** facilitates transmission, such as of programming instructions and application data, between the CPU **410**, I/O device interface **440**, storage **420**, network interface **430**, and memory **460**. CPU **410** is included to be representative of a single CPU, multiple CPUs, a single CPU having multiple processing cores, and the like. And the memory **460** is generally included to be representative of a random access memory. The storage **420** may be a disk drive storage device. Although shown as a single unit, the storage **420** may be a combination of fixed and/or removable storage devices, such as tape drives, removable memory cards or optical storage, network attached storage (NAS), or a storage area-network (SAN). Further, system **400** is included to be representative of a physical computing system as well as virtual machine instances hosted on a set of underlying physical computing systems. Further still, although shown as a single computing system, one of ordinary skill in the art will recognized that the components of the system **400** shown in FIG. **4** may be distributed across multiple computing systems connected by a data communications network.

As shown, the memory **460** includes an operating system **461** and applications **462-463**. Illustratively, the operating system may include Microsoft Windows®. The applications **462-463** include a warping application **462** and an image editing application **463**. The warping application **462** may be configured to determine content-aware BBWs given a source image **421** and warp(s) **423** from the source image **421** to the target image(s) **422**. In addition, the warping application may transfer received edits which are made to the source image **421** to the target image(s) **423** using the respective warp(s) associated with those target image(s) **423**. In one embodiment, the warping application may determine content-aware BBWs **424** and warp functions(s) **423**, and transfer edits to the source image **421**, according to the methods **200** and **300** discussed above. Illustratively, the source image **421** and target image **422**, as well as edits thereto, may be stored in the storage **420**. Similarly, the warp function(s) **423** and content-aware BBWs **424** determined by the warping application **462** may be stored in the storage **420**.

The image editing application **463** may generally be any application which permits a user to edit a source image. Illustratively, the image editing application **463** may include Adobe's Photoshop®. Although shown as separate from the warping application **462**, the image editing application **463** may also be part of the warping application **462**, or vice versa. For example, the image editing application **463** may be a component within the warping application **462** which provides an interface for users to edit the source image. Upon receiving such edits, the warping application **462** may use the appropriate warp(s) to transfer the edits to target image(s).

Although discussed above with respect to transferring image edits, content-aware BBWs possessing high-order edge-aware smoothness and additional properties, discussed above, are suitable for many other applications, such as matting, colorization, localized tonal adjustment, and image deformation.

Although discussed above primarily with respect to content-aware BBWs, other content-aware weight functions, or non-content-aware weight functions, may be used in lieu of content-aware BBWs. For example, bounded biharmonic, biharmonic, harmonic, and global weight functions may be used. Nevertheless, content-aware BBWs have properties, such as location interaction, smoothness, boundedness, and content awareness, which some other weight functions may lack. Experience has shown that, given these properties, use of content-aware BBWs may be particularly advantageous.

Although discussed above primarily with respect to warps from source to target images, the target images may be mapped back to the source image using "backwards warping." That is, warps may generally be used in both directions (i.e., to warp source image coordinates to target image coordinates, and vice versa). For example, backwards warping may include bilinearly interpolating the color values in the target coordinates and writing them back to the corresponding input source coordinates.

Advantageously, techniques disclosed herein permit edits made to a source image to be correctly transferred to target image(s) despite relatively large perspective changes, illumination changes, and non-rigid deformations between the source image and the target image(s). Further, techniques discussed herein account for relatively large occlusions in the target image(s) such that edits may be correctly transferred despite the occurrence of such occlusions. In addition, techniques discussed herein guarantee mathematical smoothness of weight functions, thereby producing continuous warps in cases where previous techniques would generate discontinuous warps.

While the foregoing is directed to aspects of the present invention, other and further aspects of the invention may be devised without departing from the basic scope thereof, and the scope thereof is determined by the claims that follow.

What is claimed is:

1. A computer-implemented method for transferring image edits from a source image to a target image, comprising:

determining, via one or more processors, content-aware weights for the source image, wherein the content-aware weights are content-aware bounded biharmonic weights (BBWs);

determining, via the one or more processors, an initial warp from the source image to the target image;

iteratively optimizing the initial warp using the content-aware weights; and

applying the optimized warp to transfer the image edits to the target image.

2. The method of claim **1**, further comprising, receiving a selection of a region of interest in the source image.

3. The method of claim **1**, further comprising,

determining gradient magnitude values for the source image;

determining a confidence image by applying heat diffusion to the gradient magnitude values; and

iteratively selecting a plurality of highest confidence locations, wherein the iteratively selecting includes subtracting a Gaussian kernel from the confidence image after each selection of a highest confidence location, and wherein the selected plurality of highest confidence locations provide seed locations for the content-aware weights.

4. The method of claim **1**, wherein determining the content-aware weights includes optimizing BBWs in a feature space.

5. The method of claim **4**, wherein initializing the warp from the source image to the target image includes determining a piecewise affine map between scale-invariant transform (SIFT) features of the target image and SIFT features of the source image, and projecting the piecewise affine map to a space of maps spanned by the degrees of freedom of the feature space.

6. The method of claim **4**, further comprising:

downsampling the source image prior to optimizing the BBWs in the feature space; and

upsampling the optimized BBWs using interpolation.

7. The method of claim **1**, wherein iteratively optimizing the warp function using the content-aware weights includes performing an inverse compositional Lucas-Kanade procedure.

8. The method of claim **7**, wherein the iterative optimization further includes optimizing per-weight bias and gain parameters.

9. A non-transitory computer-readable storage medium storing instructions, which when executed by a computer system, perform operations for transferring image edits from a source image to a target image, the operations comprising:

determining, via one or more processors, content-aware weights for the source image, wherein the content-aware weights are content-aware bounded biharmonic weights (BBWs);

determining, via the one or more processors, an initial warp from the source image to the target image;

iteratively optimizing the initial warp using the content-aware weights; and

applying the optimized warp to transfer the image edits to the target image.

10. The computer-readable storage medium of claim **9**, the operations further comprising, receiving a selection of a region of interest in the source image.

11. The computer-readable storage medium of claim **9**, the operations further comprising,

determining gradient magnitude values for the source image;

determining a confidence image by applying heat diffusion to the gradient magnitude values; and

iteratively selecting a plurality of highest confidence locations, wherein the iteratively selecting includes subtracting a Gaussian kernel from the confidence image after each selection of a highest confidence location, and wherein the selected plurality of highest confidence locations provide seed locations for the content-aware weights.

12. The computer-readable storage medium of claim **9**, wherein determining the content-aware weights includes optimizing BBWs in a feature space.

13. The computer-readable storage medium of claim **12**, wherein initializing the warp from the source image to the target image includes:

determining a piecewise affine map between scale-invariant transform (SIFT) features of the target image and SIFT features of the source image, and

projecting the piecewise affine map to a space of maps spanned by the degrees of freedom of the feature space.

14. The computer-readable storage medium of claim **12**, the operations further comprising:

downsampling the source image before optimizing the BBWs in the feature space; and

upsampling the optimized BBWs using interpolation.

15. The computer-readable storage medium of claim **9**, wherein iteratively optimizing the warp function using the content-aware weights includes performing the inverse compositional Lucas-Kanade procedure.

16. The computer-readable storage medium of claim **15**, wherein the iterative optimization further includes optimizing per-weight bias and gain parameters.

17. A system, comprising:

a processor; and

a memory, wherein the memory includes an application program configured to perform operations for transferring image edits from a source image to a target image, the operations comprising:

determining content-aware weights for the source image, wherein the content-aware weights are content-aware bounded biharmonic weights (BBWs),

determining an initial warp from the source image to the target image,

iteratively optimizing the initial warp using the content-aware weights, and

applying the optimized warp to transfer the image edits to the target image.

18. The system of claim **17**, wherein determining the content-aware weights includes optimizing BBWs in a feature space, and wherein iteratively optimizing the warp function using the content-aware weights includes performing an inverse compositional Lucas-Kanade procedure.

* * * * *

UNITED STATES PATENT AND TRADEMARK OFFICE

CERTIFICATE OF CORRECTION

PATENT NO. : 9,202,431 B2
APPLICATION NO. : 13/654335
DATED : December 1, 2015
INVENTOR(S) : Sorkine-Hornung et al.

Page 1 of 4

It is certified that error appears in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

On the Title Page

(73) Assignee:
Please delete “Disney Enterprises, Inc., Burbank, CA (US)” and insert -- Disney Enterprises, Inc., Burbank, CA (US); ETH Zurich (Eidgenoessische Technische Hochschule Zurich), Zurich (CH) -- therefor;

In the Specification

Column 4, Line 67, please delete “ $\mathcal{M}(p)=\Sigma_{k=1}{}^{m}w_k(p)T_kp$ ” and insert -- $\mathcal{M}(\mathbf{p}) = \sum_{k=1}^{m} w_k(\mathbf{p})T_k\mathbf{p}$ -- therefor;

Column 5, Line 32, please delete “ $\arg\min_{w_k,k=1,\ .\ .\ .\ ,m}\Sigma_{k=1}{}^{m}½\int_{I_S}(\Delta w_k)^2dx\,dy$ ” and insert -- $\arg\min_{w_k,\ k=1,\ldots,m} \sum_{k=1}^{m} \frac{1}{2}\int_{I_S}(\Delta w_k)^2\,dx\,dy$ -- therefor;

Column 5, Line 34, please delete “ subject to: $w_k|H_l=\delta_{kl},k,l=1,\ .\ .\ .\ ,m$ ” and insert -- subject to: $w_k|_{H_l} = \delta_{kl},\ k,l = 1,\ldots,m$ -- therefor;

Signed and Sealed this
Seventeenth Day of January, 2017

Michelle K. Lee

Michelle K. Lee
Director of the United States Patent and Trademark Office

Column 5, Line 36, please delete " $\Sigma_{k=1}{}^{m} w_k(p) - 1 \forall p \epsilon l_s$ " and insert -- $\sum_{k=1}^{m} w_k(\mathbf{p}) = 1 \quad \forall \mathbf{p} \in I_s$ -- therefor;

Column 5, Line 39, please delete " $0 \leq w_k(p) \leq 1, k = 1, \ldots, m, \forall p \epsilon l_s.$ " and insert -- $0 \leq w_k(\mathbf{p}) \leq 1,\ k = 1, \ldots, m,\ \forall \mathbf{p} \in I_s.$ -- therefor;

Column 5, Line 45, please delete " $\arg\min_{w_k} \Sigma_{k=1}{}^{m} w_k{}^T Q w_k,$ " and insert -- $\arg\min_{\mathbf{w}_k} \sum_{k=1}^{m} \mathbf{w}_k^T Q \mathbf{w}_k,$ -- therefor;

Column 5, Lines 53-57, please delete

"
$$L_{ij} = \begin{cases} -\frac{1}{2}(\cot\alpha_{ij} + \cot\alpha_{ji}) & \text{if } j \in N(i) \\ -\sum_{l \in N(i)} L_{il} & \text{if } j = 1 \\ 0 & \text{otherwise} \end{cases}$$
" and

insert --
$$L_{ij} = \begin{cases} -\frac{1}{2}\left(\cot\alpha_{ij} + \cot\alpha_{ji}\right) & \text{if } j \in N(i) \\ -\sum_{l \in N(i)} L_{il} & \text{if } j = 1 \\ 0 & \text{otherwise} \end{cases}$$
-- therefor;

Column 8, Lines 7-8, please delete

" $\arg\min_{T_k{}^0, k=1, \ldots, m} \Sigma_{p \epsilon \tau} r(p) \| \mathcal{M}_{SIFT}(p) - \Sigma_{i=1}{}^{m} w_k(p) T_k{}^0 p \|^2,$ " and

insert -- $\arg\min_{T_k^0,\ k=1,\ldots,m} \sum_{\mathbf{p} \in \tau} r(\mathbf{p}) \left\| \mathcal{M}_{\mathrm{SIFT}}(\mathbf{p}) - \sum_{i=1}^{m} w_k(\mathbf{p}) T_k^0 \mathbf{p} \right\|^2,$ -- therefor;

Column 8, Line 19, please delete " $\gamma \Sigma_{p \epsilon l_s} r(p) \| Ap - \Sigma_{i=1}{}^{m} w_k(p) T_k{}^0 p \|^2,$ "

and insert -- $\gamma \sum_{\mathbf{p} \in I_s} r(\mathbf{p}) \left\| A\mathbf{p} - \sum_{i=1}^{m} w_k(\mathbf{p}) T_k^0 \mathbf{p} \right\|^2,$ -- therefor;

Column 8, Line 20, please delete "T" and insert -- τ -- therefor;

Column 8, Line 43, please delete

“ $\arg\min \mathcal{M}\, \Sigma_{p \in I_s} r(p)\phi(I_t(\mathcal{M}(p)), I_s(p)).$ ” and

insert -- $\arg\min_{\mathcal{M}} \sum_{\mathbf{p}\in I_s} r(\mathbf{p})\phi\left(I_t(\mathcal{M}(\mathbf{p})), I_s(\mathbf{p})\right)$ -- therefor;

Column 8, Line 63, please delete

“ $\arg\min_{T_k, k=1, \ldots, m} \Sigma_{p \in I_s} r(p)\phi(I_t(\Sigma_{i=1}{}^{m} w_k(p) T_k p), I_s(p))$ ”

and insert -- $\arg\min_{T_k,\ k=1,\ldots,m} \sum_{\mathbf{p}\in I_s} r(\mathbf{p})\phi\left(I_t(\sum_{i=1}^{m} w_k(\mathbf{p}) T_k \mathbf{p}), I_s(\mathbf{p})\right)$ -- therefor;

Column 9, Line 16, please delete

“ $\mathcal{M}(p) \leftarrow [\Sigma_{k=1}{}^{m} w_k(p) T_k p] / [\Sigma_{l=1}{}^{m} w_l(p) \Delta T_l p]^{-1},$ ”

and insert -- $\mathcal{M}(\mathbf{p}) \leftarrow [\sum_{k=1}^{m} w_k(\mathbf{p}) T_k \mathbf{p}][\sum_{l=1}^{m} w_l(\mathbf{p}) \Delta T_l \mathbf{p}]^{-1},$ -- therefor;

Column 9, Line 20, please delete

“ $=\Sigma_{k=1}{}^{m} [w_k(p) T_k [\Sigma_{l=1}{}^{m} w_l(p) T_l]^{-1}] p.$ ” and

insert -- $= \sum_{k=1}^{m} [w_k(\mathbf{p}) T_k [\sum_{l=1}^{m} w_l(\mathbf{p}) T_l]^{-1}]\, \mathbf{p}.$ -- therefor;

Column 9, Line 27, please delete

“ $\approx\Sigma_{k=1}{}^{m} [w_k(p) T_k [\Sigma_{l=1}{}^{m} w_l(p) \Delta T_k]^{-1}] p.$ ” and

insert -- $\approx \sum_{k=1}^{m} [w_k(\mathbf{p}) T_k [\sum_{l=1}^{m} w_l(\mathbf{p}) \Delta T_k]^{-1}]\, \mathbf{p}.$ -- therefor;

Column 9, Line 33, please delete “∀l≠k: w_l(p)≈0” and insert -- $\forall l \neq k$: $w_l(\mathbf{p}) \approx 0$ -- therefor;

Column 9, Line 34, please delete “l≠k” and insert -- $l \neq k$ -- therefor;

Column 9, Line 35, please delete “l=k” and insert -- $l = k$ -- therefor;

Column 9, Line 53, please delete

“ $\mathcal{M}(p) \leftarrow \Sigma_{k=1}{}^{m} [w_k(p) T_k [\Delta T_k]^{-1}] p.$ ” and

insert -- $\mathcal{M}(\mathbf{p}) \leftarrow \sum_{k=1}^{m} [w_k(\mathbf{p}) T_k [\Delta T_k]^{-1}]\, \mathbf{p}.$ -- therefor;

Column 10, Line 6, please delete "$I_s(p)+\Sigma_{k=1}{}^{m} w_k(p)(a_k I_s(p)+b_k),$"

and insert -- $I_s(\mathbf{p}) + \sum_{k=1}^{m} w_k(\mathbf{p})(a_k I_s(\mathbf{p}) + b_k),$ -- therefor.